Contribution ID : **24** Type : **not specified**

Differential Equations and Nonlinear Natural Phenomena

Isaac Newton, one of the founders of the theory of differential equations, was also one of the first scholars to realize its great significance for the study of natural laws. Indeed, since a couple of centuries ago one cannot imagine physics, biology, economics and many other fields without differential equations which are the universal language for the scientists. Despite this, the theory of nonlinear differential equations is relatively new. In the middle of the XX century important research work by E. Fermi, J. Pasta, and S. Ulam took place, which led to the discovery of a beautiful phenomenon called *soliton*, and hence the nonlinear theory got a huge impulse for the development. It further appeared that the solitons play a crucial role in various fields of science: biology (a neural impulse), hydrodynamics (solitary waves, wave envelopes), thermodynamics (thermal impulse) and many other.

To our mind, the best way to start dealing with nonlinear dynamical systems is to approach them by utilizing the numerical methods. In fact, the abovementioned soliton was discovered as a numerical solution to the celebrated *Korteweg - de Vries (KdV) equation* [1]
$\mathrm{u}_t + u_{xxx} - 6uu_x = 0.$

Therefore, our first acquaintance with nonlinear partial differential equations (PDE's) was the numerical investigation of soliton collisions for *the nonlinear Schroedinger equation* [1] i $\psi_t + \frac{1}{2}\psi_{xx} + \kappa|\psi|^2\psi = 0,$

which is of a critical significance in natural science. As a result of a number of simulations, different configurations of nonlinear waves interactions were analyzed. It was also empirically confirmed, that the solitons for this equation behave as particles.

On the other hand, for the complete picture of nonlinear phenomena, the numerical approach does not suffice. Indeed, after the important discovery by Zabuski and Kruskal, more and more scientists dedicated their studies to the integrability theory, conservation laws and finding exact solutions to nonlinear PDE's. Unfortunately, there still is no uniform approach to these problems, and hence a plenty of work to do.

In view of all this, our further works were focussed on an analytical investigation of some nonlinear equations of mathematical physics. As a result, by means of the (G'/G) - expansion method, a number of new exact solutions to some KdV - type equations, in particular, the soliton ones, were obtained [2]. On the other hand, the main objective of our current studies are the conservation laws and the integrability analysis of some inverse nonlinear dynamical systems to explain a nature and an internal structure of these equations.

To sum up, we are convinced that nowadays the progress in the natural sciences (in particular, in nonlinear problems) strongly relies on theoretical results of mathematics. On the other hand, applications and modeling in physics and biology not only verify the theory but also provide new problems for mathematicians. In our opinion, only the effective cooperation between mathematics and natural sciences makes possible the further exploration of Nature.

References

1. *Ablowitz, M., Sigur, H.*, 1982. Solitary Waves and the Inverse Problem Method. Mir, Moscow.
2. *Mykhailiuk, I., Prytula, M.*, 2017. Application of (G'/G) - Expansion Method to Two Korteweg - de Vries Type Dynamic Systems. Journal of Numerical and Applied Mathematics, [Online]. 3 (126), 86 - 103. Available at: http://jnam.lnu.edu.ua/pdf/y2017_no3(126)_art07_mykhailiuk_prytula.pdf.

Summary

Primary author(s) : MYKHAILIUK, Ivan (Ivan Franko National University of Lviv)

Presenter(s) : MYKHAILIUK, Ivan (Ivan Franko National University of Lviv)

Session Classification : Poster Session